

Correlation between Confinement and Chiral-Symmetry Breaking in the Dual Higgs Theory

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Abstract

We study the correlation between confinement and dynamical chiral-symmetry breaking ($D\chi SB$) in the dual Ginzburg-Landau (DGL) theory using the effective potential formalism. The DGL theory is an infrared effective theory based on the dual Higgs mechanism, and provides the nonperturbative gluon propagator, which leads to the linear quark potential. The screening effect for the confining potential can be obtained by introducing the infrared cutoff corresponding to the hadron size. We formulate the effective potential for $D\chi SB$ in the DGL theory, and find the vacuum instability against quark condensation. To extract confinement effect, we separate the effective potential into the confinement part and others by dividing the confinement term from other terms in the gluon propagator in the DGL theory. The confinement part provides the dominant contribution to $D\chi SB$, which is regarded as monopole dominance for $D\chi SB$. The relevant energy for $D\chi SB$ is found to be the infrared region below 1GeV. Monopole dominance for the DGL propagator is also found in the intermediate region, $0.2\text{fm} \lesssim r \lesssim 1\text{fm}$.

1 Introduction

Quark confinement and dynamical chiral-symmetry breaking ($D\chi SB$) are the most important nonperturbative phenomena in quantum chromodynamics (QCD) [1]. Color confinement is characterized by the formation of the squeezed color-electric flux tube [2] and the linear potential between quark and anti-quark [3, 4], which confines the quarks inside hadrons. On the other hand, $D\chi SB$ is characterized by quark condensate, large effective quark mass and the appearance of the Nambu-Goldstone (NG) bosons as pions [5]. Although the QCD Lagrangian is chirally symmetric in the massless quark limit, it is spontaneously broken in the nonperturbative vacuum due to the quark condensate $\langle \bar{q}q \rangle \neq 0$. Such phenomena concerning with $D\chi SB$ has been demonstrated and studied by using the effective models of QCD [5, 6, 7, 8], for instance Nambu-Jona-Lasinio model [9]. These studies suggest that light quarks get a large effective mass [10], $M \simeq 350\text{MeV}$, in the infrared energy region as a result of $D\chi SB$. The absence of the parity doubling in the hadronic spectrum is understood in terms of $D\chi SB$, and the pion properties are understood by its identification to the the NG boson. As for confinement, it is getting clear that quark confinement can be understood in terms of the dual superconductor picture in the 't Hooft abelian gauge [11] from recent studies of the lattice gauge theory [12, 13, 14]. In this picture, the QCD-monopole appearing

by the abelian gauge fixing condenses in the QCD vacuum, and therefore quarks are confined through the dual Meissner effect [15, 16, 17].

The close relation between these two nonperturbative phenomena [18] have been suggested by the coincidence between the deconfinement phase transition and the chiral restoration in the lattice QCD [19]. Within the effective theory approach, we have shown that QCD-monopole condensation plays an essential role on $D\chi SB$ [20, 21, 22, 23, 24]. Independently, Miyamura [25] and Woloshyn [26] found the similar result of *monopole dominance for $D\chi SB$* in the lattice gauge theory. The DGL theory is an infrared effective theory of the nonperturbative QCD, where the QCD vacuum is described in terms of the abelian gauge theory with QCD-monopoles [27]. In this theory, it was shown that monopole condensation induces large dynamical quark mass using the Schwinger-Dyson(SD) equation, a non-linear integral equation for the quark mass function [20, 21, 22]. The chiral phase transition at finite temperature was also studied by solving the SD equation. A strong correlation was found between the string tension and the critical temperature of the chiral symmetry restoration in terms of QCD-monopole condensation [23]. Thus, the above results suggest strong correlation between confinement and $D\chi SB$.

To get deeper insight on this correlation, it is desirable to examine the contribution of confinement to $D\chi SB$ more directly. In the DGL theory, the nonperturbative gluon propagator is composed of two parts, confinement term and others. As for the inter quark potential, the former leads to the linear potential and latter to the Yukawa potential. To examine the contribution of each part, we calculate the effective potential in the ladder level approximation, where we can simply extract the contribution of confinement to $D\chi SB$. In Sec.2, we review the description of confinement in the DGL theory. We derive the nonperturbative gluon propagator from the DGL Lagrangian, and see how it reproduces the $q-\bar{q}$ confining potential. In Sec.3, we formulate the effective potential in terms of the quark propagator. The effective potential is formulated in the ladder level approximation with the QCD renormalization improvement for the gauge coupling. An analysis of the nonperturbative gluon propagator is done also. In Sec.4, numerical results are discussed. We also compare our results with the QCD-like theory, where the perturbative gluon propagator is used in the ladder approximation. Sec.5 is devoted to summary and concluding remarks.

2 Dual Higgs Mechanism and Quark Confinement

It was pointed out that the quantum chromodynamics(QCD), a non-abelian gauge theory, reduces to an abelian gauge theory containing monopoles in the 't Hooft abelian gauge [11], which is defined by diagonalizing an appropriate gauge-dependent variable $X[A_\mu(x)]$. Since this gauge fixing condition leaves abelian gauge degrees of freedom unfixed, the theory holds the local symmetry on the abelian gauge transformation. In this gauge, only the diagonal components of the gluon behave as abelian gauge fields and the remaining off diagonal components behave as charged matter fields under the residual abelian gauge transformation. One of the most important features of the 't Hooft abelian gauge is that monopoles appear at the singular points in gauge fixing condition [11].

From the recent analyses based on the lattice QCD, the remaining off-diagonal gluon is largely reduced in the maximally abelian (MA) gauge [28] and nonperturbative features are almost reproduced only by the abelian gluon including QCD-monopoles [13, 25, 26]. For instance, in the SU(2) lattice gauge theory, MA gauge is defined by maximizing

$$R = \sum_{s, \hat{\mu}} \text{Tr}[\sigma_3 U_\mu(s) \sigma_3 U_\mu^\dagger(s)] \quad (1)$$

by the gauge transformation. Here, $U_\mu(s) \equiv e^{iagA_\mu(s)}$ denotes the lattice link variable. In this gauge, $X[A_\mu(x)]$ to be diagonalized is given by the Lorentz scalar,

$$X(s) = \sum_{\mu} [U_\mu(s)\sigma_3 U_\mu^\dagger(s) + U_\mu^\dagger(s-\mu)\sigma_3 U_\mu(s-\mu)]. \quad (2)$$

In the continuum limit, the MA gauge fixing is equivalent to $(\partial^\mu \pm iaA_3^\mu)A_\mu^\pm = 0$, where A_3^μ and A_\pm^μ denote the diagonal gluon and the off-diagonal gluons, respectively. In the lattice formalism, the SU(2) link-variable in MA gauge is factorized as $U_\mu = e^{i(\theta_\mu^+ T^- + \theta_\mu^- T^+)} e^{i\theta_\mu^3 T^3}$, where the abelian link-variable $u_\mu \equiv e^{i\theta_\mu^3 T^3}$ corresponds to the abelian part of U_μ . Similarly to the Wilson loop, the abelian Wilson loop, defined by the loop integral $w = e^{i \oint ds^\mu \theta_\mu^3 T^3}$, obeys the area law and reproduces the string tension well in MA gauge [13]. Therefore, the only abelian gluon seems relevant in MA gauge for confinement in the lattice QCD, which is called as abelian dominance for confinement. Thus, for the description of the confinement physics, QCD in MA gauge would be well approximated by the abelian gauge theory including monopoles.

The abelian gauge field coupled to electric current j_μ and magnetic current k_μ is described by introducing the dual gauge field B_μ as [29]

$$\mathcal{L}_{\text{Zwanziger}} = K_{\text{gauge}}(A, B) - j_\mu A^\mu - k_\mu B^\mu. \quad (3)$$

The kinetic term of gauge fields $K_{\text{gauge}}(A_\mu, B_\mu)$ is expressed in the Zwanziger form [21, 27],

$$K_{\text{gauge}}(A_\mu, B_\mu) \equiv -\frac{2}{n^2} [n \cdot (\partial \wedge A)]^\nu [n \cdot {}^* (\partial \wedge B)]_\nu - \frac{1}{n^2} [n \cdot (\partial \wedge A)]^2 - \frac{1}{n^2} [n \cdot (\partial \wedge B)]^2, \quad (4)$$

which manifests the duality of the gauge theory. Here, n_μ is an arbitrary constant space-like 4-vector corresponding to the Dirac string direction. The diagonal gauge field A_μ and the dual gauge field B_μ are defined on the Cartan sub-algebra $\vec{H} = (T_3, T_8)$: $A^\mu \equiv A_3^\mu T_3 + A_8^\mu T_8$, $B^\mu \equiv B_3^\mu T_3 + B_8^\mu T_8$.

The dual Ginzburg-Landau (DGL) theory is an infrared effective theory of QCD based on the dual Higgs mechanism in the abelian gauge [20, 21, 22, 27, 30, 31, 32]. Its Lagrangian is described by quarks q , monopoles χ , and diagonal part of gauge fields as [32]

$$\mathcal{L}_{\text{DGL}} = \text{tr} K_{\text{gauge}}(A_\mu, B_\mu) + \bar{q}(i \not{\partial} - e \not{A} - m)q + \text{tr} [\mathcal{D}_\mu, \chi]^\dagger [\mathcal{D}^\mu, \chi] - \lambda \text{tr} (\chi^\dagger \chi - v^2)^2, \quad (5)$$

where $\mathcal{D}_\mu \equiv \partial_\mu + igB_\mu$ is the dual covariant derivative with the dual gauge coupling g obeying the Dirac condition, $eg = 4\pi$ [21]. The QCD-monopole field χ is defined on the nontrivial root vectors E_α : $\chi \equiv \sqrt{2} \sum_{\alpha=1}^3 \chi_\alpha E_\alpha$, with $\sqrt{2}E_1 \equiv T_1 + iT_2$, $\sqrt{2}E_2 \equiv T_4 - iT_5$, $\sqrt{2}E_3 \equiv T_6 + iT_7$. The QCD-monopoles condense in the vacuum, due to its self interaction. Its condensation squeezes out the color electric flux and leads to linear inter-quark potential [21, 22, 27].

Let us derive the nonperturbative gluon propagator in the QCD-monopole condensed vacuum [21]. At the mean field level for the monopole condensate $|\chi_\alpha| = v$, one obtains the Lagrangian density as

$$\mathcal{L}_{\text{MF}} = \text{tr} K_{\text{gauge}}(A_\mu, B_\mu) + \bar{q}(i \not{\partial} - e \not{A} - m)q + \frac{1}{2} m_B^2 \vec{B}^2, \quad (6)$$

where B_μ acquires a mass of $m_B = \sqrt{3}gv$. By integrating out the dual gauge field B_μ , one obtains the nonperturbative gluon propagator for A_μ

$$D_{\mu\nu}(k) = -\frac{1}{k^2} \{g_{\mu\nu} + (\alpha_e - 1) \frac{k_\mu k_\nu}{k^2}\} - \frac{1}{k^2} \frac{m_B^2}{k^2 - m_B^2} \frac{1}{(n \cdot k)^2} \epsilon^{\lambda \mu \alpha \beta} \epsilon^{\lambda \nu \gamma \delta} n^\alpha n^\gamma k^\beta k^\delta, \quad (7)$$

with the gauge parameter α_e . The double pole $1/(n \cdot k)^2$ in the second term causes the infrared linear part in the static quark-antiquark potential. Hence, the second term is regarded as the nonperturbative part of the gluon propagator. The above expression, however, is derived without taking into account the dynamics of light quarks. With the dynamical quarks in the vacuum, the flux tube between static charges when the distance exceeds some typical length as hadron size, will be cut into two tubes with pair-produced quarks on its end. As a result, the static potential will be screened in the QCD vacuum. For the study of chiral symmetry, which is a symmetry of light quarks, we have to take into account the screening effect by the light quark polarization [21]. In this case, the nonperturbative part is modified with the infrared cutoff parameter a corresponding to the inverse of hadron size as [22]

$$D_{\mu\nu}(k) = -\frac{1}{k^2} \{g_{\mu\nu} + (\alpha_e - 1) \frac{k_\mu k_\nu}{k^2}\} - \frac{1}{k^2} \frac{m_B^2}{k^2 - m_B^2} \frac{1}{(n \cdot k)^2 + a^2} \epsilon^{\lambda \mu \alpha \beta} \epsilon_{\lambda \nu \gamma \delta} n^\alpha n^\gamma k^\beta k^\delta. \quad (8)$$

Now we derive the static inter-quark potential. Introducing external color-electric source \vec{j} , the current action can be obtained as

$$\begin{aligned} Z_j &= \int \mathcal{D}A e^{i \int d^4x \mathcal{L}_{\text{MF}}(A, j)} = \int \mathcal{D}A \exp \left\{ i \int d^4x \left[\frac{1}{2} \vec{A}^\mu(x) D_{\mu\nu}^{-1}(x) \vec{A}^\nu(x) - \vec{j}_\mu(x) \vec{A}^\mu(x) \right] \right\} \\ &= \exp \left\{ i \int d^4x \int d^4y \left[-\frac{1}{2} \vec{j}^\mu(x) D_{\mu\nu}(x-y) \vec{j}^\nu(y) \right] \right\} \equiv e^{-iV[j]} \int dt. \end{aligned} \quad (9)$$

For the static quark and anti-quark current

$$\vec{j}^\mu(x) = \vec{Q} g^{\mu 0} [\delta^3(\mathbf{x} - \mathbf{b}) - \delta^3(\mathbf{x} - \mathbf{a})], \quad (10)$$

whose Fourier component is given as

$$\vec{j}^\mu(k) = \vec{Q} g^{\mu 0} 2\pi \delta(k^0) (e^{-i\mathbf{k} \cdot \mathbf{b}} - e^{-i\mathbf{k} \cdot \mathbf{a}}), \quad (11)$$

the current action S_j is expressed as

$$\begin{aligned} S_j &= -\frac{1}{2} \int d^4x d^4y \vec{j}^\mu(x) \cdot D_{\mu\nu}(x-y) \vec{j}^\nu(y) \\ &= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \vec{j}^\mu(-k) \cdot \left[\frac{1}{k^2 - m_B^2} g_{\mu\nu} + \left\{ \frac{m_B^2 - a^2}{k^2 - m_B^2} + \frac{a^2}{k^2} \right\} \frac{1}{(n \cdot k)^2 + a^2} g_{\mu\nu} \right] \vec{j}^\nu(k) \\ &= -\vec{Q}^2 \int dt \int \frac{d^3\mathbf{k}}{(2\pi)^3} (1 - \cos \mathbf{k} \cdot \mathbf{r}) \left[\frac{1}{\mathbf{k}^2 + m_B^2} + \frac{m_B^2(\mathbf{k}^2 + a^2)}{\mathbf{k}^2(\mathbf{k}^2 + m_B^2)} \frac{1}{(\mathbf{n} \cdot \mathbf{k})^2 + a^2} \right]. \end{aligned} \quad (12)$$

Here, we have used $n^\mu = (0, \mathbf{n})$ for the Dirac string direction with unit vector \mathbf{n} and the relative coordinate $\mathbf{r} = \mathbf{b} - \mathbf{a}$. The first term produces the short-range Yukawa potential while the second term gives rise to the infrared linear potential.

We set $\mathbf{n} \parallel \mathbf{r}$ from the symmetry of the system and the energy minimum condition. Similarly in the argument for the Abrikosov vortex, there appears the physical ultraviolet cutoff corresponding to the coherent length in the integration of the transverse momentum k_T [21], due to the reduction of the monopole condensate in the core region of the flux tube. Quantitatively, the inverse of the coherent length is given by the monopole mass $m_\chi = 2\sqrt{\lambda}v$, and the dual gluon mass m_B vanishes in the core region $r_T \lesssim m_\chi^{-1}$, which leads to the cutoff on the ultraviolet region $k_T \gtrsim m_\chi$ in the above integral. Thus, the static potential is obtained as

$$V(r) = -\frac{\vec{Q}^2}{4\pi} \frac{e^{-m_B r}}{r} + \sigma \frac{1 - e^{-ar}}{a}, \quad (13)$$

with string tension given as

$$\sigma = \frac{\vec{Q}^2}{8\pi} \left\{ (m_B^2 - a^2) \ln \left(\frac{m_\chi^2 + m_B^2 - a^2}{m_B^2 - a^2} \right) + a^2 \ln \left(\frac{m_\chi^2 - a^2}{a^2} \right) \right\}. \quad (14)$$

The second term of Eq.(13) reduces to the linear potential σr in the limit of $a \rightarrow 0$, which corresponds to the absence of the screening effects. In the region $r \gtrsim a^{-1}$, screening effect suppress the potential and it approaches to a constant value in the limit $r \rightarrow \infty$. In Fig.1, we show the static potential in the DGL theory for various infrared screening cutoff a together with the phenomenological Cornell potential [33],

$$V_{\text{Cornell}}(r) = -\frac{e_C^2}{3\pi} \cdot \frac{1}{r} + k_C r, \quad (15)$$

with $e_C \simeq 2$ and $k_C \simeq 1\text{GeV}/\text{fm}$. The DGL parameter set is to be chosen so as to reproduce the string tension $\sigma = 1\text{GeV}/\text{fm}$, and the flux-tube radius of hadrons as $m_B^{-1} = 0.3 \sim 0.4$ fm. Here, we set $m_B = 0.5\text{GeV}$, $e = 5.5$, and choose $m_\chi = 1.26, 1.17$ and 1.06 GeV for $a = 0, 85, 150$ MeV respectively. In the recent lattice QCD simulation [34], the screening effect by dynamical quarks is observed at the infrared region as $r \gtrsim 2\text{fm}$, which corresponds to $a \lesssim 100$ MeV.

There are two possibilities of modification on the short range part [31]. One is the vanishing of the dual gauge mass m_B and the other is the running of the gauge coupling. Though the Yukawa type part is obtained by integrating the first term of the Eq.(12) without cutoff, m_B should vanish in the ultraviolet region as noted above. This may cause the shift of the Yukawa type potential to a Coulomb type one in the short range. Also, in this region, the gauge coupling constant is expected to be modified. In the DGL theory, the off diagonal gluons are omitted on the basis of abelian dominance. However, this feature is expected only for the long range physics and in the perturbative region all components of the gluon become relevant. Hence, with the increase of the number of coupling gluons, the description only with abelian components becomes insufficient. To compensate the contribution of off diagonal components, the coupling constant should be modified in this region. Considering these modifications one may find better coincidence between the static potential presented by DGL theory and the phenomenological one in the short range.

In any case, the dual Ginzburg–Landau theory describes the global feature of the confinement dynamics well. In addition, as stated in the introduction, this model exhibits $D\chi\text{SB}$ for quarks as well. Therefore one can inquire much about these nonperturbative physics through this model, instead of dealing with the highly complex full QCD in the infrared region.

3 Correlation between $D\chi\text{SB}$ and Confinement

Recently, the essential role of QCD-monopole condensation on $D\chi\text{SB}$ has been pointed out by solving the Schwinger-Dyson (SD) equation [20, 21, 22]. In this chapter, we study the correlation between $D\chi\text{SB}$ and confinement from the view point of energy density, *i.e.* in the effective potential formalism. Approximating the full quark propagator as $iS^{-1} = \not{p} - M(p^2) + i\epsilon$, one obtains the SD equation for the quark mass function $M(p^2)$ in the Euclidean space,

$$M(p^2) = \int \frac{d^4q}{(2\pi)^4} \vec{Q}^2 \frac{M(q^2)}{q^2 + M^2(q^2)} D_{\mu\mu}(q-p) \quad (16)$$

in the chiral limit. Here, $\vec{Q}^2 = \frac{N_c-1}{2N_c}e^2$ denotes the abelian electric charge of the quark. In this equation, $D_{\mu\mu}(k)$ is composed of three parts,

$$D_{\mu\mu}(k) = \frac{2m_B^2}{k^2(k^2 + m_B^2)} \cdot \frac{n^2k^2 + a^2}{(n \cdot k)^2 + a^2} + \frac{2}{k^2 + m_B^2} + \frac{1 + \alpha_e}{k^2} = D_{\mu\mu}^{\text{conf}}(k) + D_{\mu\mu}^{\text{Y}}(k) + D_{\mu\mu}^{\text{C}}(k). \quad (17)$$

The first term $D_{\mu\mu}^{\text{conf}}(k)$ is responsible for the linear confinement potential [20, 21] in the absence of the screening effect, $a = 0$. The second term $D_{\mu\mu}^{\text{Y}}(k)$ is related to the short-range Yukawa potential, and the Coulomb part $D_{\mu\mu}^{\text{C}}(k)$ does not contribute to the quark static potential. In spite of this decomposition, it is difficult to separate and compare each contribution to $D\chi\text{SB}$ in the nonlinear SD equation. To examine these contributions separately, it is useful to study the effective potential which corresponds to the vacuum energy density. In this paper, we study the effective potential [35] variationally in the chiral limit.

Within the ladder level approximation, the effective potential $V_{\text{eff}}[S]$ leads to the ladder SD equation when imposed the extremum condition in terms of the full quark propagator $S(p)$ [7]. Using the nonperturbative gluon propagator $D^{\mu\nu}(k)$ in the DGL theory, the effective potential as a functional of the dynamical quark mass $M(p^2)$ is expressed as

$$V_{\text{eff}}[S] = i\text{Tr} \ln[S_0 S^{-1}] + i\text{Tr}[S S_0^{-1}] + \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} \frac{\vec{Q}^2}{2} \text{tr}[\gamma_\mu S(p) \gamma_\nu S(q) D^{\mu\nu}(p-q)], \quad (18)$$

where $S_0(p)$ is the bare quark propagator, $iS_0^{-1}(p) = \not{p} - m + i\epsilon$.

Here, we would like to comment on the wavefunction renormalization. Considering the effect of the wavefunction renormalization, the ladder SD equation for the quark propagator, $iS^{-1}(p) = z^{-1}(p^2)(\not{p} - M(p^2))$, is given by coupled equations

$$M(p^2) = z(p^2) \int \frac{d^4 q}{(2\pi)^4 i} \vec{Q}^2 z(q^2) \frac{M(q^2) g^{\mu\nu}}{q^2 - M^2(q^2)} D_{\mu\nu}, \quad (19)$$

$$z^{-1}(p^2) = 1 - \frac{1}{p^2} \int \frac{d^4 q}{(2\pi)^4 i} \vec{Q}^2 z(q^2) \frac{p^\mu q^\nu + p^\nu q^\mu - p \cdot q g^{\mu\nu}}{q^2 - M^2(q^2)} D_{\mu\nu}. \quad (20)$$

Suppose one uses the perturbative gluon propagator and the Higashijima–Miransky approximation [7, 8] for the running coupling constant

$$e^2(k^2) \simeq e^2(\bar{k}^2), \quad (21)$$

where $k = p - q$, $\bar{k}^2 \equiv \max(p^2, q^2)$. Then, Eq.(20) reduces to the form

$$z^{-1}(p^2) = 1 + \alpha_e \frac{2}{p^2} \int \frac{d^4 q}{(2\pi)^4 i} \vec{Q}^2(\bar{k}^2) \frac{z(q^2)}{q^2 - M^2(q^2)} \frac{p \cdot q}{(p-q)^2}, \quad (22)$$

which leads $z(p^2) \equiv 1$ in the Landau gauge ($\alpha_e = 0$). In this case, Eq.(19) reduces to Eq.(16). On the other hand, with the use of the nonperturbative gluon propagator in Eq.(8), this is not necessarily the case. In the Landau gauge, however, we found $z(p^2) \simeq 1$ even in the DGL theory by solving the coupled SD equations including $z(p^2)$ [36].

Hence, working in the Landau gauge, we approximate the wavefunction renormalization as $z(p^2) \simeq 1$, which corresponds to the neglect of the second term in the right hand side of Eq.(20).

In the effective potential formalism, its 2-loop term is given as

$$\begin{aligned}
V^{(2)} &= 2N_f N_c \int \frac{d^4 q}{(2\pi)^4 i} \int \frac{d^4 p}{(2\pi)^4 i} \bar{Q}^2 z(q^2) z(p^2) \frac{M(q^2)M(p^2)g^{\mu\nu} + p^\mu q^\nu + p^\nu q^\mu - p \cdot q g^{\mu\nu}}{[M^2(q^2) - q^2][M^2(p^2) - p^2]} D_{\mu\nu} \\
&= 2N_f N_c \int \frac{d^4 q}{(2\pi)^4 i} \int \frac{d^4 p}{(2\pi)^4 i} \bar{Q}^2 z(q^2) z(p^2) \frac{M(q^2)M(p^2)}{[M^2(q^2) - q^2][M^2(p^2) - p^2]} D^\mu{}_\mu \\
&+ 2N_f N_c \int \frac{d^4 p}{(2\pi)^4 i} \frac{p^2}{M^2(p^2) - p^2} \{1 - z(p^2)\}, \tag{23}
\end{aligned}$$

where Eq.(20) is used. Here, to set the effect of the wavefunction renormalization as $z(p^2) \equiv 1$ is equivalent to drop off the second term of Eq.(23). This approximation would hold good in the Landau gauge because the coupled SD equations shows $z(p^2) \equiv 1$ [36].

Hence, the effective potential consistent with the approximation $z(p^2) \equiv 1$ is given by

$$\begin{aligned}
V_{\text{eff}}[M(p^2)] &\equiv V_{\text{quark}}[M(p^2)] + V_{\text{q-g}}[M(p^2)] \\
&= -2N_f N_c \int \frac{d^4 p}{(2\pi)^4} \left\{ \ln\left(\frac{p^2 + M^2(p^2)}{p^2}\right) - 2\frac{M^2(p^2)}{p^2 + M^2(p^2)} \right\} \\
&- N_f(N_c - 1) \int \frac{d^4 p}{(2\pi)^4} \frac{d^4 q}{(2\pi)^4} e^2 \frac{M(p^2)}{p^2 + M^2(p^2)} \frac{M(q^2)}{q^2 + M^2(q^2)} D_{\mu\mu}(p - q) \tag{24}
\end{aligned}$$

in the Euclidean space. It is easy to check that the extremum condition on Eq.(24) with respect to $M(p^2)$ leads to the SD equation (16) [21, 22] exactly.

In Eq.(24), the first term V_{quark} is the one-loop contribution as in Fig.2a. The second term $V_{\text{q-g}}$ with $D_{\mu\mu}$ is the two-loop contribution with the quark-gluon interaction as expressed in Fig.2b. The important point is that this second term $V_{\text{q-g}}$ is divided into three parts as

$$V_{\text{q-g}} = V_{\text{conf}} + V_Y + V_C, \tag{25}$$

corresponding to the decomposition of $D_{\mu\mu}$ in Eq.(17). Hence, by estimating V_{conf} , V_Y , V_C respectively, it is possible to examine each contribution to $D\chi\text{SB}$.

Before going further, remember that the nonperturbative gluon propagator depends on the Dirac string direction n_μ . However, due to the $q\bar{q}$ pair polarization effect around the quark, the Dirac-string direction n_μ becomes indefinite [22]. Therefore we take the average value on n_μ ,

$$\left\langle \frac{n_\mu n_\nu}{(n \cdot k)^2 + a^2} \right\rangle_{\text{av}} = \frac{1}{2\pi^2} \int d\Omega_n \frac{n_\mu n_\nu}{(n \cdot k)^2 + a^2} = \frac{k_\mu k_\nu}{k^2} f_{\parallel}(k^2) + \left\{ \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right\} f_{\perp}(k^2), \tag{26}$$

with

$$f_{\parallel}(k^2) = \frac{1}{(a + \sqrt{a^2 + k^2})^2}, \tag{27}$$

$$f_{\perp}(k^2) = \frac{a + 2\sqrt{a^2 + k^2}}{3a(a + \sqrt{a^2 + k^2})^2} = \frac{1}{3} \left\{ \frac{2(a^2 + k^2)}{ak^2(a + \sqrt{a^2 + k^2})} - \frac{1}{k^2} \right\}, \tag{28}$$

where the angle integration is performed in the Euclidean 4-dimensional space. We then obtain the averaged gluon propagator $\bar{D}_{\mu\nu}$ as

$$\bar{D}_{\mu\nu} = \frac{1}{3} \left\{ \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right\} \left\{ \frac{1}{k^2} + \frac{2}{k^2 + m_B^2} + \frac{4m_B^2}{k^2(k^2 + m_B^2)} \frac{a^2 + k^2}{a(a + \sqrt{a^2 + k^2})} \right\} + \alpha_e \frac{k_\mu k_\nu}{(k^2)^2} \tag{29}$$

which requires the modification to Eq.(17) only on confinement part as

$$D_{\mu\mu}^{\text{conf}} \rightarrow \frac{4m_B^2}{k^2(k^2 + m_B^2)} \frac{k^2 + a^2}{a(a + \sqrt{k^2 + a^2})}. \quad (30)$$

Therefore the effective potential we ought to compute is given as the sum of following four terms

$$V_{\text{quark}} = \frac{2N_f N_c}{(4\pi)^2} \int_0^\infty dp^2 \left\{ -p^2 \ln \left(1 + \frac{M^2(p^2)}{p^2} \right) + \frac{2p^2 M^2(p^2)}{M^2(p^2) + p^2} \right\} \quad (31)$$

$$V_C = -\frac{2N_f N_c}{(4\pi)^4} \int_0^\infty dp^2 \frac{p^2 M(p^2)}{M^2(p^2) + p^2} \int_0^\infty dq^2 \frac{q^2 M(q^2)}{M^2(q^2) + q^2} \bar{Q}^2 \frac{1}{\max(p^2, q^2)} \quad (32)$$

$$V_Y = -\frac{2N_f N_c}{(4\pi)^4} \int_0^\infty dp^2 \frac{p^2 M(p^2)}{M^2(p^2) + p^2} \int_0^\infty dq^2 \frac{q^2 M(q^2)}{M^2(q^2) + q^2} \bar{Q}^2 \\ \times \frac{4}{\pi} \int_0^\pi d\theta \sin^2 \theta \frac{1}{(p-q)^2 + m_B^2} \quad (33)$$

$$V_{\text{conf}} = -\frac{2N_f N_c}{(4\pi)^4} \int_0^\infty dp^2 \frac{p^2 M(p^2)}{M^2(p^2) + p^2} \int_0^\infty dq^2 \frac{q^2 M(q^2)}{M^2(q^2) + q^2} \bar{Q}^2 \\ \times \frac{8}{\pi a} \int_0^\pi d\theta \sin^2 \theta \frac{1}{a + \sqrt{(p-q)^2 + a^2}} \frac{m_B^2 ((p-q)^2 + a^2)}{(p-q)^2 ((p-q)^2 + m_B^2)}. \quad (34)$$

As for the running coupling, we adopt hybrid type one [21, 22] in the Higashijima–Miransky approximation [7, 8],

$$e^2((p-q)^2) = \frac{48\pi^2(N_c + 1)}{11N_c - 2N_f} \left[\ln \frac{p_c^2 + \max(p^2, q^2)}{\Lambda_{\text{QCD}}^2} \right]^{-1}. \quad (35)$$

Choosing p_c as

$$p_c^2 = \Lambda_{\text{QCD}}^2 \exp\left[\frac{48\pi^2}{e^2} \frac{N_c + 1}{11N_c - 2N_f} \right], \quad (36)$$

the above expression connects smoothly the perturbative running coupling of QCD in the ultraviolet region and the infrared effective coupling e of the DGL theory [21, 22]. From the renormalization group analysis of QCD [7], the approximate form of the quark-mass function $M(p^2)$ is expected as

$$M(p^2) = M(0) \frac{p_c^2}{(p_c^2 + p^2)} \left[\ln \frac{p_c^2}{\Lambda_{\text{QCD}}^2} \Big/ \ln \frac{p_c^2 + p^2}{\Lambda_{\text{QCD}}^2} \right]^{1 - \frac{N_c^2 - 1}{2N_c} \cdot \frac{9}{11N_c - 2N_f}}. \quad (37)$$

Since the exact solution $M_{\text{SD}}(p^2)$ of the SD equation (16) [22] is well reproduced by this ansatz (37) with $M(0) \simeq 0.4\text{GeV}$ and $p_c^2 \simeq 10\Lambda_{\text{QCD}}^2$, we use this form as a variational function of the effective potential.

At the end of this section, let us examine the nonperturbative gluon propagator in the DGL theory. By performing the Fourier transformation on Eq.(29), we show in Fig.3 the Coulomb, Yukawa and confinement parts of the DGL gluon propagator in the coordinate space,

$$D_{\mu\mu}^{\text{C}}(x-y) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2} e^{ik(x-y)} = \frac{1}{4\pi^2} \frac{1}{(x-y)^2} \quad (38)$$

$$D_{\mu\mu}^Y(x-y) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{2}{k^2 + m_B^2} e^{ik(x-y)} \quad (39)$$

$$D_{\mu\mu}^{\text{conf}}(x-y) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{4m_B^2}{k^2(k^2 + m_B^2)} \frac{a^2 + k^2}{a(a + \sqrt{a^2 + k^2})} e^{ik(x-y)} \quad (40)$$

as the function of $r \equiv |x-y|$. The confinement part gives a dominant contribution to the DGL propagator in the whole region, especially $0.2\text{fm} \lesssim r \lesssim 1\text{fm}$. This means that a strong correlation is brought by the confinement part of the nonperturbative gluon propagator. Since this confinement part is a direct consequence of the monopole condensation, this significant contribution suggests *monopole dominance for the DGL propagator*, at least in the intermediate region.

4 Numerical Results and Discussions

We show in Fig.4 the effective potential V_{eff} as a function of the infrared effective quark mass $M(0)$, using the mass function (37) with $p_c^2 = 10\Lambda_{\text{QCD}}^2$. We have used the same parameters as in Ref. [21, 22], $\lambda = 25$, $v = 0.126\text{GeV}$, $e = 5.5$ and $a = 85\text{MeV}$ so as to reproduce the inter-quark potential and the flux-tube radius $R \simeq 0.4\text{fm}$ [21]. It takes a minimum at finite $M(0) \simeq 0.4\text{GeV}$, which means that the nontrivial solution is more stable than the trivial one in terms of the energetical argument. Hence, chiral symmetry is spontaneously broken.

We show in Fig.5 $V_{\text{quark}}, V_{\text{conf}}, V_Y$ and V_C as the function of $M(0)$. The lowering of the effective potential contributes to $D\chi\text{SB}$. Although both the Yukawa and the Coulomb terms contribute slightly to lower the effective potential, it is mainly lowered by the confinement part V_{conf} and there is a large cancellation between V_{quark} and V_{conf} . Thus, $D\chi\text{SB}$ is brought by V_{conf} arising from monopole condensation in the DGL theory which is regarded as monopole dominance for $D\chi\text{SB}$. Such a dominant role of the confinement effect on $D\chi\text{SB}$ is found for any value of $M(0)$.

In Fig.6, we show also the integrands of each term

$$V_{\text{eff}} = \int_0^\infty dp^2 v_{\text{eff}}(p^2) = \int_0^\infty dp^2 \{v_{\text{quark}}(p^2) + v_{\text{conf}}(p^2) + v_Y(p^2) + v_C(p^2)\} \quad (41)$$

to examine which energy region is important to $D\chi\text{SB}$. Here we have used the exact solution $M_{\text{SD}}(p^2)$ of the SD equation (16) [21, 22] as the mass function to get rid of the ambiguity arising from the choice of the trial mass function. Among v_{conf}, v_Y and v_C the confinement part v_{conf} is always dominant for all momentum region. All the three terms, v_{conf}, v_Y , and v_C , contribute to the effective potential mainly in the low momentum region less than 1GeV although there are also long tails running into high momentum region over 1GeV . These behaviors directly reflect the profile of the quark mass function $M_{\text{SD}}(p^2)$. It is notable that such contributions from high momentum region are strongly canceled in v_{eff} as shown in Fig.6. Consequently, the remaining contribution only from low momentum region ($p^2 < 0.4\text{GeV}^2$) plays an important role to $D\chi\text{SB}$. This seems consistent with the above result that infrared confinement effect is dominant for $D\chi\text{SB}$.

Now, we compare our results with those in the QCD-like theory [7, 8, 37, 38], which is a more familiar framework for $D\chi\text{SB}$. In the QCD-like theory, the perturbative gluon propagator without the confinement force is used and the effective potential V_{eff} can be divided into two parts, V_{quark} and $V_{\text{q-g}}$ within the improved ladder approximation [7, 8]. Here, V_{quark} is given by Eq.(31), and $V_{\text{q-g}}$ takes a simple form [7],

$$V_{\text{q-g}} = \int_0^\infty dp^2 v_{\text{q-g}}(p^2) = -\frac{2N_f N_c C}{(4\pi)^4} \int_0^\infty dp^2 \frac{p^2 M(p^2)}{M^2(p^2) + p^2} \int_0^\infty dq^2 \frac{q^2 M(q^2)}{M^2(q^2) + q^2} \frac{3e^2(\max(p^2, q^2))}{\max(p^2, q^2)} \quad (42)$$

with $C = \frac{N_c^2 - 1}{2N_c}$ and the running coupling constant

$$e^2(p^2) = \frac{48\pi^2}{11N_c - 2N_f} \left\{ \ln \frac{p_c^2 + p^2}{\Lambda_{\text{QCD}}^2} \right\}^{-1}. \quad (43)$$

The effective potential as a function of the infrared quark mass in the QCD-like theory also takes the double-well structure when the gauge coupling constant is enough large [7]. Similarly in the DGL theory, the driving force of D χ SB is brought by the quark-gluon interaction part $V_{\text{q-g}}$, and a large cancellation between V_{quark} and $V_{\text{q-g}}$ is found there.

As for the relevant energy scale for D χ SB, we show in Fig.7 the integrands $v_{\text{quark}}(p^2)$ and $v_{\text{q-g}}(p^2)$ using the solution of the SD equation in the QCD-like theory with parameters, $e(0) = 11$ and $\Lambda_{\text{QCD}} = 746\text{MeV}$, which reproduce $f_\pi = 93\text{MeV}$ and $\langle \bar{q}q \rangle_{\text{RGI}} = -(239\text{MeV})^3$ [38]. Again the relevant energy scale is found to be less than 0.5 GeV for D χ SB, which suggests the importance of the treatment on the infrared strong-coupling region, where large confinement effects would appear. In the DGL theory, the confining force has been included as a result of monopole condensation, which has been supported in the lattice gauge theory [12, 13, 14], and therefore the DGL approach seems to provide a more consistent picture for the infrared region in QCD.

In the QCD-like theory, large values for $e(0)$ and Λ_{QCD} are often used to reproduce f_π or the constituent quark mass [37, 38]. As an interesting possibility, such an ‘‘enhancement’’ of the strong coupling region may be regarded as a compensation for the neglected confinement effect in the QCD-like theory [21]. In any case, it would be meaningful to include some confinement effect in the QCD-like theory to argue whether the confinement effect is relevant to D χ SB in QCD [39].

5 Summary and Concluding Remarks

We have studied confinement and dynamical chiral-symmetry breaking (D χ SB) in the dual Ginzburg–Landau (DGL) theory using the effective potential formalism. The DGL theory describes confinement by squeezing of the color-electric flux through dual Higgs mechanism. The resulting gluon propagator is composed of two parts, one is the usual perturbative part and the other nonperturbative term leading to the linear potential between static q - \bar{q} system. To investigate the correlation between D χ SB and confinement we have formulated the effective potential for the quark propagator. Making use of the nonperturbative gluon propagator in the DGL theory we have included the effect of confinement into the effective potential. Within the ladder approximation with the renormalization group improvement for the coupling, the effective potential is formulated as a function of the dynamical quark mass $M(p^2)$.

The effective potential has been calculated as a function of the infrared quark mass $M(0)$ with the variational function suggested by renormalization group analysis of QCD. We have found the double-well structure of the effective potential, so that the nontrivial solution is more stable than the trivial one and leads to D χ SB. To examine the role of confinement, the interaction term $V_{\text{q-g}}$ has been divided into the confinement part V_{conf} and others (V_Y, V_C). We have found that the confinement part V_{conf} stemming from monopole condensation gives the dominant contribution to $V_{\text{q-g}}$ ($V_{\text{q-g}} \simeq V_{\text{conf}}$), which means the *monopole dominance* for D χ SB. It has been also found that the low momentum contribution from less than 1 GeV plays an important role for D χ SB.

Finally, we would like to consider the critical scale between the nonperturbative QCD (NP-QCD) and the perturbative QCD (P-QCD) in terms of the dual Higgs mechanism [40].

In this framework, these regions are divided by the typical energy scale of QCD-monopole condensation, $m_B(m_\chi) \sim 1\text{GeV}$. Above the energy scale, the system can be characterized by the asymptotic freedom in P-QCD with the Coulomb interaction. On the other hand, the infrared strong-coupling region would be characterized by monopole condensation in the dual Higgs theory, because QCD-monopoles seem to play relevant role to the nonperturbative phenomena such as confinement and $D\chi\text{SB}$. It is interesting to compare this critical scale with a *normalization point* μ of the operator product expansion (OPE) in the QCD sum rule [41], where the quantum fluctuations with the momentum scale above μ are included in the Wilson coefficient, while those with the momentum scale below μ are included in the local operator. Since the short-scale physics are approximately described by the P-QCD while the long-scale physics are connected to nonperturbative phenomena, μ physically corresponds to the border of NP-QCD and P-QCD. Hence, the scale of QCD-monopole condensation, $m_B(m_\chi) \sim 1\text{GeV}$, may provide the physical image of μ in OPE. In any case, we conjecture that QCD in the 't Hooft abelian gauge exhibits the two different theoretical aspects: one is the short-distance physics subject to P-QCD formulated in the trivial vacuum, and the other is the infrared nonperturbative physics described by the dual Higgs theory with its QCD-monopole condensation.

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Figure Captions

Fig.1: Static $q-\bar{q}$ potential in the DGL theory. Phenomenological potential (dashed curve) is also shown for comparison. With the increase of the screening cutoff a , the screening effect becomes strong. The saturation of the potential is observed at the long distance, $r \gtrsim 1/a$. The phenomenological potential is also shown for comparison.

Fig.2: The diagrams which contribute to the effective potential in the ladder level approximation. (a) The quark loop contribution V_{quark} without the explicit interaction. (b) The two-loop diagram $V_{\text{q-g}}$ including the quark-gluon interaction. Here, the curly line with a black dot denotes the nonperturbative gluon propagator in the DGL theory.

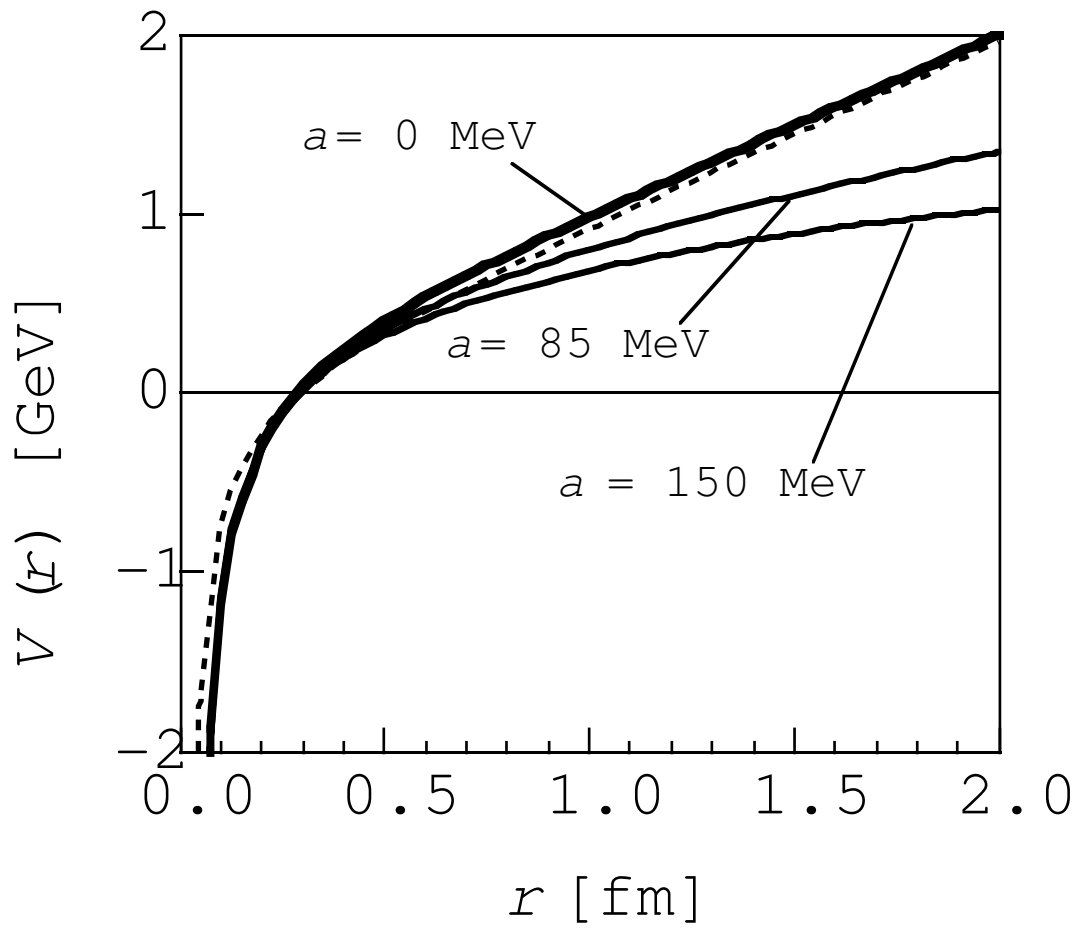
Fig.3: The DGL gluon propagator in the coordinate space, $D_{\mu\mu}(r) \equiv F.T.D_{\mu\mu}(k)$, with $m_B = 0.5\text{GeV}$ and $a = 85\text{MeV}$. The linear part is significantly large compared to the other parts especially in the intermediate region $0.2\text{fm} \lesssim r \lesssim 1\text{fm}$. Thus, the long-range strong interaction is mainly brought by the confinement part in the DGL theory.

Fig.4: The total effective potential V_{eff} as a function of the infrared quark mass $M(0)$. The nontrivial minimum appears at $M(0) \sim 0.4\text{GeV}$, which indicates dynamical breaking of chiral symmetry.

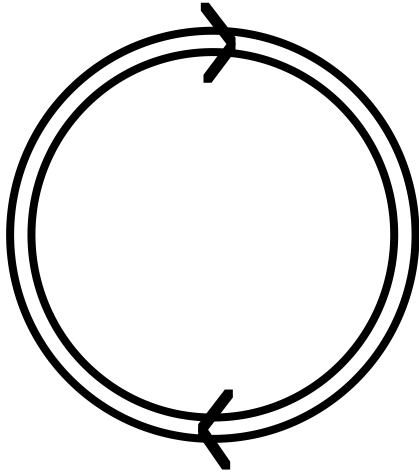
Fig.5: V_{quark} , V_{conf} , V_Y and V_C are shown as a function of $M(0)$. The confinement part V_{conf} , plays the dominant role through the lowering the effective potential.

Fig.6: Integrands v_{quark} , v_{conf} , v_Y and v_C of effective potential are shown as functions of the Euclidean momentum p^2 . The confinement part v_{conf} is more significant than v_Y and v_C for all momentum region.

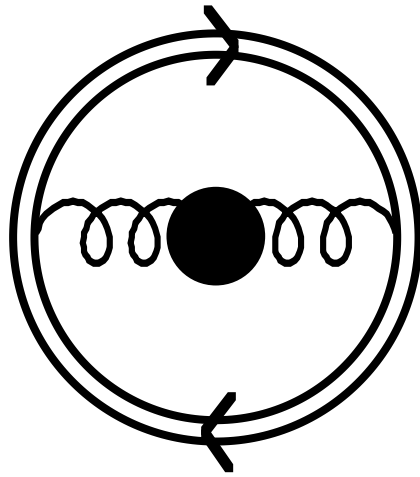
Fig.7: The integrands v_{eff} , v_{quark} and $v_{\text{q-g}}$ in the QCD-like theory. These quantities are plotted as functions of the Euclidean momentum squared p^2 .



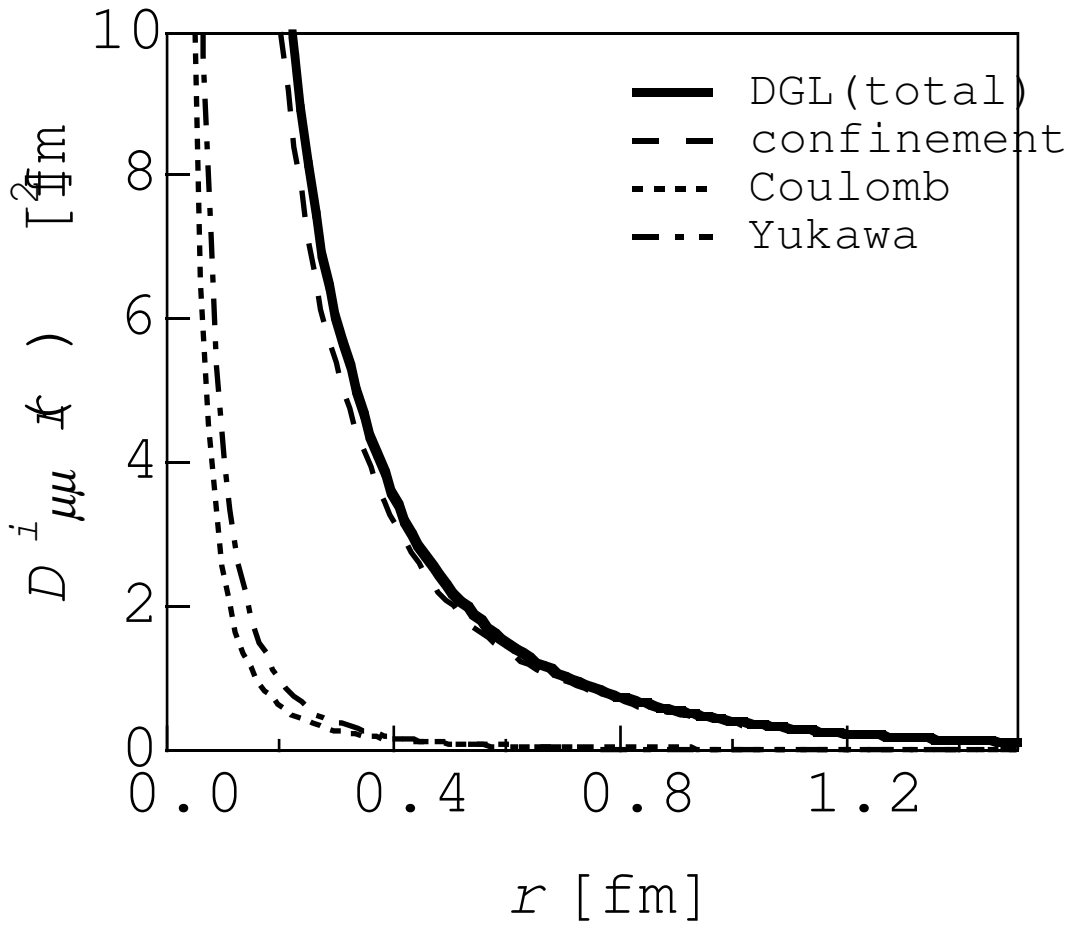
PRD.Fig.1 S. Umisedo et al.



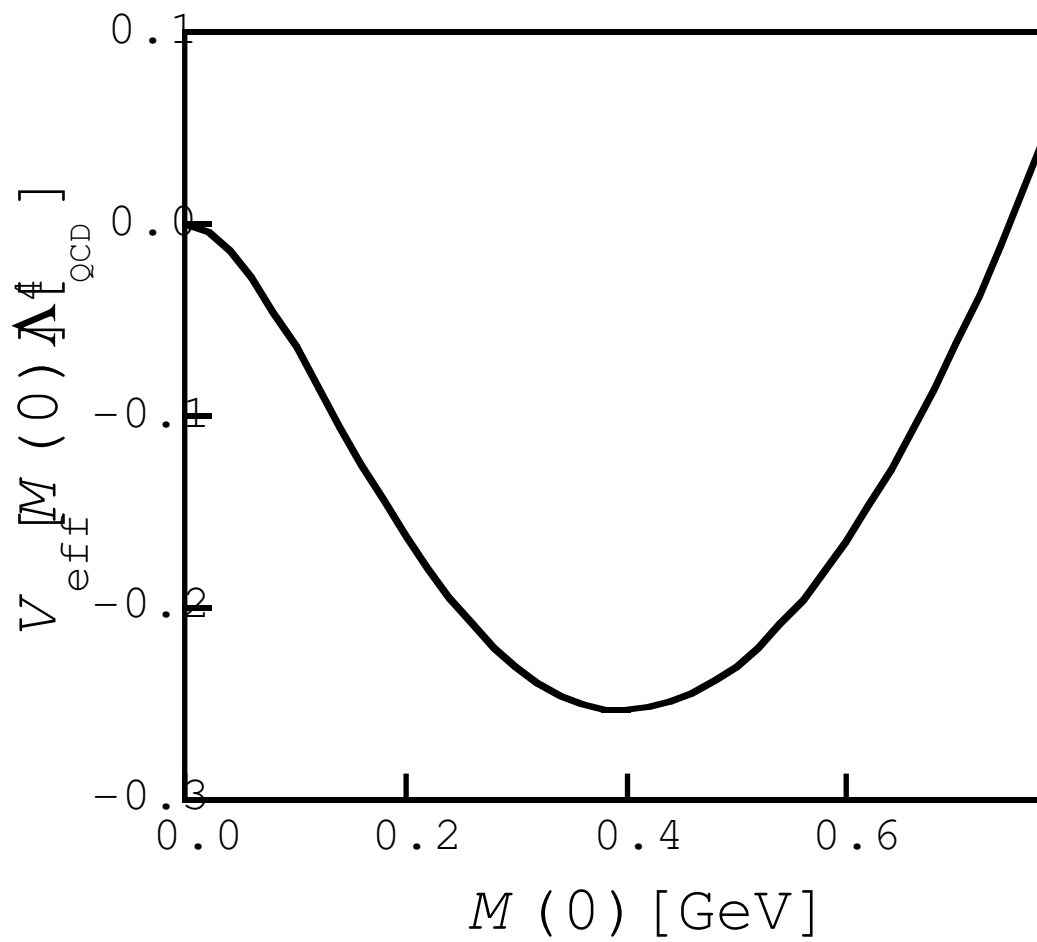
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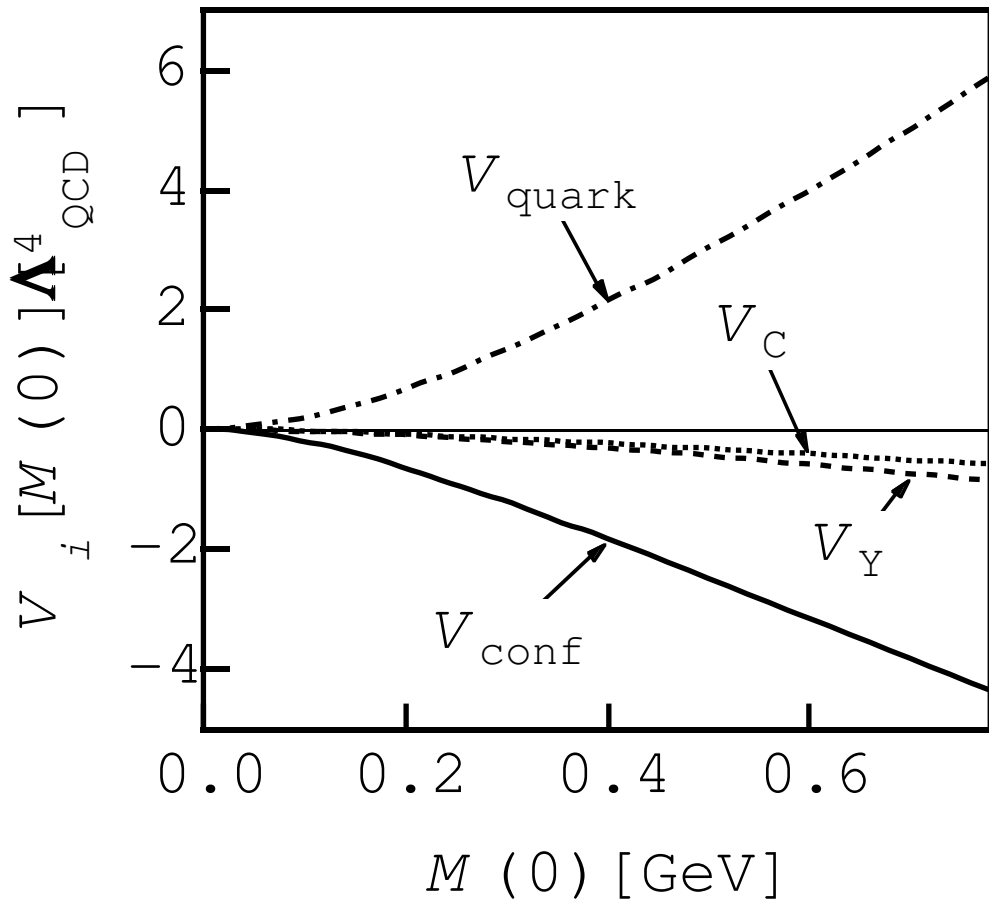
PRD.Fig.2b S. Umisedo et al.



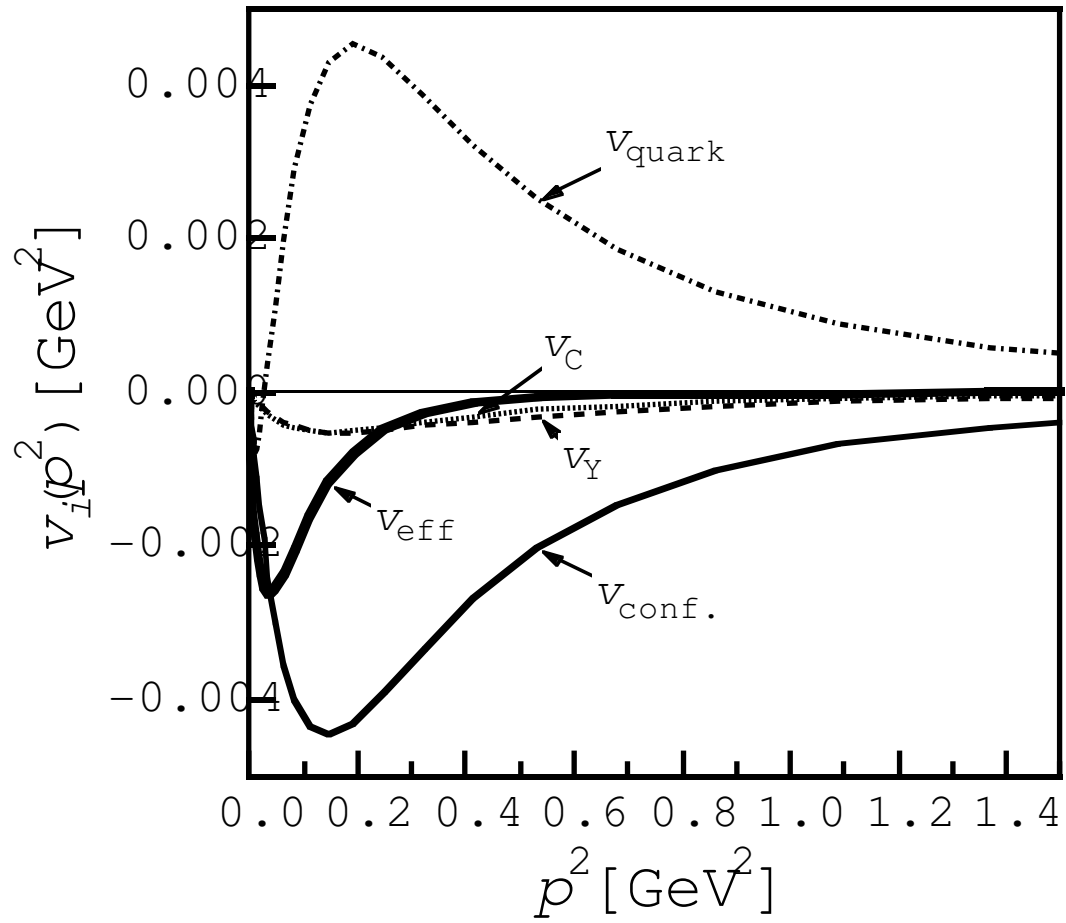
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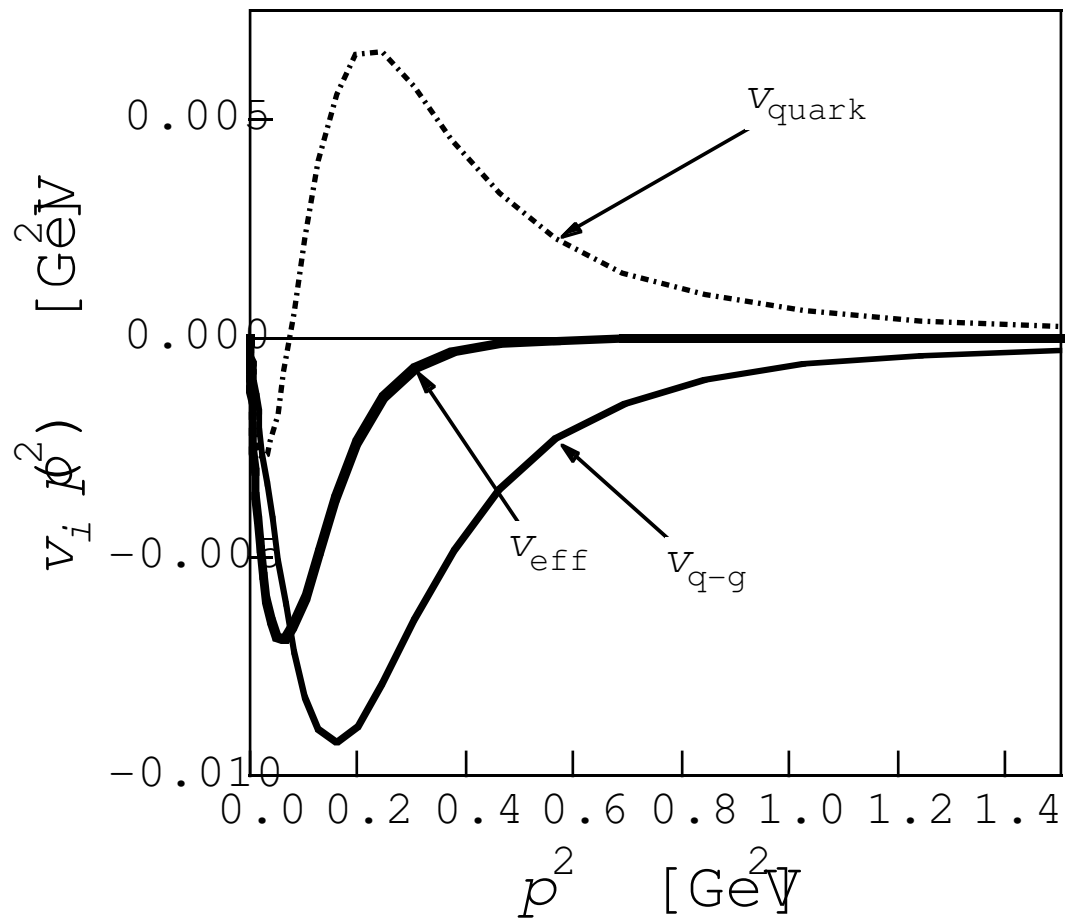
PRD.Fig.4 S. Umisedo et al.



PRD.Fig.5 S. Umisedo et al.



PRD.Fig6. S. Umisedo et al.



PRD.Fig.7 S. Umisedo et al.